Jeezik Optimization Model

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1 Problem

Anesthesiologists must be scheduled to various different departments on any given day. The challenge for the scheduler is to (1) identify a feasible schedule that (2) minimizes total travel time and (3) maintains an equitable workload week-to-week.

The typical solution is for an anesthesiologist to create the schedule manually which may lead to inequitable workloads and unnecessary travel.

2 Solution

The proposed model only aims to (1) identify a feasible schedule that (2) minimizes total travel time.

It does not consider an equitable workload day-to-day or week-to-week.

It is sufficient to demonstrate my ability to model and develop a linear programming problem consisting of a variable number of decision variables.

3 Model

3.1 Definitions

$$
\begin{aligned}
G &= \text{The set of days in the schedule} \\
H_g &= \text{The set of anesthesiologists available to work on day g} \in \text{G} \\
R &= \text{The set of real nodes} \\
S &= \text{The set containing the source dummy node} \\
D &= \text{The set containing the destination dummy node} \\
I,\ J &= R \cup S \cup D \\
c_{gij} &= \text{The cost of traveling from node i to node j on day g} \\
n_g &= \text{The number of nodes on day g} \\
b_g &= \begin{cases} n_g, \text{ if } n_g < |H_g| \ \ \forall g \\ H_g, \text{ otherwise} \end{cases}
\end{aligned}
$$

3.2 Decision Variables

$$
x_{ghij} = \begin{cases} 1, \text{ if on day } g \in G \text{ anesthesiologist } h \in H_g \text{ travels from node } i \in I \text{ to node } j \in J \\ 0, \text{ otherwise} \end{cases}
$$

3.3 Objective

$$
\text{Minimize} \quad z = \sum_{g \in G} \sum_{h \in H_g} \sum_{i \in I} \sum_{j \in J} c_{gij} x_{ghij}
$$

3.4 Constraints

$$\sum_{h\in H_g}\sum_{i\in I}\sum_{\substack{j\in J\\ j\neq i}} x_{ghij} = n_g + b_g \quad \forall g\in G \quad \text{(Set the total number of edges)}$$

$$\sum_{h\in H_g}\sum_{j\in R} x_{ghij} = b_g \quad \forall g\in G,\, i\in S \quad \text{(Set the number of edges from the source dummy to the real nodes)}$$

$$\sum_{h\in H_g}\sum_{i\in R} x_{ghij} = b_g \quad \forall g\in G,\, j\in D \quad \text{(Set the number of edges going from real nodes to the dummy destination)}$$

$$\sum_{h\in H_g}\sum_{i\in R}\sum_{\substack{j\in R\\ j\neq i}} x_{ghij} = n_g - b_g \quad \forall g\in G \quad \text{(Set the number of edges from real nodes to real nodes)}$$

$$\sum_{k\in R\cup D} x_{ghjk} - \sum_{i\in R\cup S} x_{ghij} = 0 \quad \forall g\in G,\, h\in H_g,\, j\in R \quad \text{(A real destination must also be a real source)}$$

$$\sum_{i\in R} x_{ghij} \leq 1 \quad \forall g\in G,\, h\in H_g,\, j\in D \quad \text{(Each doctor can go to the dummy destination at most once)}$$

$$\sum_{h\in H_g}\sum_{i\in R\cup S} x_{ghij} = 1 \quad \forall g\in G,\, j\in R \quad \text{(Each real node must be visited)}$$